

Coherent dynamics of domain formation in the Bose Ferromagnet

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We present a theory to describe domain formation observed very recently in a quenched ^{87}Rb gas, a typical ferromagnetic spinor Bose system. An overlap factor is introduced to characterize the symmetry breaking of $M_F = \pm 1$ components for the $F = 1$ ferromagnetic condensate. We demonstrate that the domain formation is a co-effect of the quantum coherence and the thermal relaxation. A thermally enhanced quantum-oscillation is observed during the dynamical process of the domain formation. And the spatial separation of domains leads to significant decay of the $M_F = 0$ component fraction in an initial $M_F = 0$ condensate.

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Very recently, the Berkeley group observed spontaneous symmetry breaking in ^{87}Rb spinor condensates [1]. Ferromagnetic domains and domain walls were clearly shown using an in-situ phase-contrast imaging. This appears the first image of the domain structure in a Bose ferromagnet. Although ferromagnetism has been intensively studied in the context of condensed matter physics and is regarded as one of the best understood phenomena in nature [2], the description of ferromagnetism is not yet complete. The conventional ferromagnets being considered are usually comprised of either classical particles (insulating ferromagnets) or fermions (itinerant ferromagnets) while Bose systems are seldom touched [3]. The realization of cold spinor ^{87}Rb gases [4], a typical ferromagnetic Bose system, has provided an opportunity to study Bose ferromagnets and thus opens up a way to a comprehensive understanding of ferromagnetism in all kinds of condensed matters.

The ferromagnetic spinor Bose gas has attracted numerous theoretical interests [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15]. On one side, researchers expect that it will show some general properties as conventional ferromagnets do. Ho [5], Ohmi and Machida [6] pointed out that this system has a spontaneous symmetry-broken ground state and a normal spin-wave excitations spectrum at small wave vector \mathbf{k} , $\omega_s = c_s k^2$. On the other side, researchers aim at exploring distinct features of the system. Studies on thermodynamics and phase transitions have revealed that the ferromagnetic spinor Bose gas displays a quite surprising phase diagram. Its Curie point can be larger by magnitudes than the energy scale of the ferromagnetic interaction between bosons, and never below the Bose-Einstein condensation point [7, 8]. It means that once the Bose gas condenses, it is already spontaneously magnetized.

A conventional ferromagnet usually has some domain structure below the Curie point, as illustrated in Fig. 1a. But whether it is true for a Bose ferromagnet is still some-

what controversial. One even questions whether there exists a Curie point in ferromagnetic Bose gases [9, 10], as the cold atomic gas under experimental conditions is usually not in the thermodynamic limit while the phase diagram mentioned above is derived from the thermal-equilibrium grand canonical ensemble [7, 8]. Nevertheless, a number of theoretical works have discussed the possibility of the domain formation [3, 13, 14, 15]. Within the mean-field theory, Zhang *et al.* found out that the ferromagnetic condensate has a dynamical instability leading to spontaneous domain formation in an initially magnetized Bose gas [13]. Moreover, Mur-Petit *et al.* showed that a multi-spin-domain structure manifests in ^{87}Rb condensates at finite temperatures [14]. The Berkeley experiment confirms that the Bose ferromagnet can indeed form domain structures at least under certain conditions [1]. Then new questions come, is the process of the domain formation similar to that inside a conventional ferromagnet, and how does the domain formation affect spin dynamics? We attempt to answer these questions in the present letter.

We start with the Hamiltonian for an $F = 1$ system in the following form [5]

$$H = \int d^3r \left[\frac{\hbar^2}{2m} \nabla \psi_a^\dagger \cdot \nabla \psi_a - (\mu - U) \psi_a^\dagger \psi_a + \frac{g_0}{2} \psi_a^\dagger \psi_{a'}^\dagger \psi_{a'} \psi_a + \frac{g_2}{2} \psi_a^\dagger \psi_{a'}^\dagger \mathbf{F}_{ab} \cdot \mathbf{F}_{a'b'} \psi_{b'} \psi_b \right], \quad (1)$$

where $\psi_a(\mathbf{r})$ is the field annihilation operator for an atom in state $M_F = a$ at point \mathbf{r} , μ is the chemical potential and U is the trapping potential. g_0 and g_2 are the spin-independent and spin-dependent mean-field interaction, respectively. For macroscopically occupied Bose systems it is common to replace the field annihilation operator for the a th spin component by its expectation value, i.e. $\varphi_a(\mathbf{r}, t) \equiv \langle \psi_a(\mathbf{r}, t) \rangle$, which for spinor condensates is con-

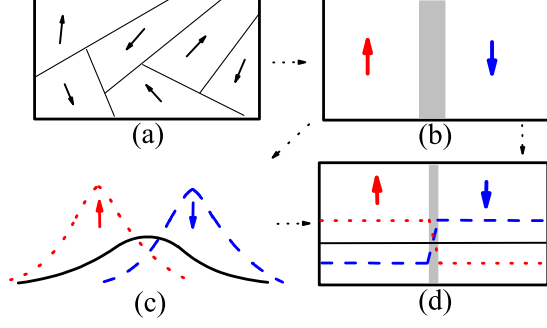


FIG. 1: Schematic domain structure inside a Ferromagnet. (a) The bulk material shows no magnetism because domains are randomly oriented. (b) The multi-domain configuration is often simplified to a two-domain structure with opposite sign of magnetization for theoretical convenience. The gray region denotes the domain wall. (c) Two-domain structure for a ferromagnetic spinor Bose-Einstein condensate. Dotted, dashed and solid lines represent the normalized particle distributions of $M_F = 1$, $M_F = -1$ and $M_F = 0$ bosons respectively. (d) In a homogeneous system, the normalized particle distribution is taken as a constant in each domain; for $M_F = 0$ bosons it remains a constant through the whole system.

veniently expressed as

$$\varphi_a(\mathbf{r}, t) = \sqrt{N_a(t)\eta_a(\mathbf{r}, t)}e^{i\phi_a(\mathbf{r}, t)}. \quad (2)$$

Here $N_a(t)$ is the number of condensed particles, $\eta_a(\mathbf{r}, t)$ denotes the normalized particle distribution with $\int d^3r \eta_a = 1$ and $\phi_a(\mathbf{r}, t)$ the phase. The multiplier of N_a and η_a refers to the condensed particle density of the a -component, and the total particle density is $\rho(\mathbf{r}, t) = N_a(t)\eta_a(\mathbf{r}, t)$. We suppose that the particle distribution can be different for distinct components in our approach. Neglecting excitations, the Hamiltonian is simplified to $H = H_0 + H_u + H_s$, with

$$\begin{aligned} H_0 &= \int d^3r \left[\frac{\hbar^2}{2m} N_a (\nabla \sqrt{\eta_a})^2 + \frac{\hbar^2}{2m} N_a \eta_a (\nabla \phi_a)^2 \right], \\ H_u &= \int d^3r \left[(U - \mu) N_a \eta_a + \frac{g_0}{2} N_a \eta_a N_{a'} \eta_{a'} \right], \\ H_s &= \int d^3r \frac{g_2}{2} \left[N_+^2 \eta_+^2 + N_-^2 \eta_-^2 - 2N_+ N_- \eta_+ \eta_- \right. \\ &\quad \left. + 2N_0 \eta_0 (N_+ \eta_+ + N_- \eta_-) \right. \\ &\quad \left. + 4N_0 \sqrt{N_+ N_-} \eta_0 \sqrt{\eta_+ \eta_-} \cos \theta \right], \end{aligned} \quad (3)$$

where $\theta = \phi_+ + \phi_- - 2\phi_0$ is the relative phase.

In case that the ground state of the condensate is symmetry-broken, certain magnetic domain structure is formed spontaneously. Within each domain the atomic magnetic moments are aligned in a preferential direction. A direct consequence of domain formation is that the $M_F = 1$ and $M_F = -1$ components are spatially separated, as portrayed in Fig. 1c. The integral

$\int d^3r \eta_+ \eta_-$ measures the extent of overlap between the two, which is called the overlap factor hereinafter. In general, several “overlap factors” should be introduced to Eqs.(3), e.g., $\alpha_{0\pm} = V \int d^3r \eta_0^2$, $\alpha_1 = V \int d^3r \eta_+ \eta_-$, $\alpha_{2\pm} = V \int d^3r \eta_0 \eta_{\pm}$ and $\alpha_3 = V \int d^3r \eta_0 \sqrt{\eta_+ \eta_-}$, where V is the volume of the system. Treating the relative phase θ as a spatially independent constant as previous theory did [11, 12, 13, 14, 15], the term of H_s in Eqs. (3) is rewritten as

$$\begin{aligned} H_s &= \frac{g_2}{2V} \left[\alpha_{0+} N_+^2 + \alpha_{0-} N_-^2 - 2\alpha_1 N_+ N_- \right. \\ &\quad \left. + 2N_0 (\alpha_{2+} N_+ + \alpha_{2-} N_-) \right. \\ &\quad \left. + 4\alpha_3 N_0 \sqrt{N_+ N_-} \cos \theta \right]. \end{aligned} \quad (4)$$

According to their definition, these overlap factors are not totally independent from each other and the number of independent ones can be further reduced. For simplicity, we consider a homogeneous spinor Bose gas with a two-domain structure, as shown in Fig. 1d, and neglect the domain wall. In this case, we derive, after some integration and algebraic manipulation, that there is only one independent overlap factor and the above equation is reduced to

$$\begin{aligned} H_s &= \frac{g_2}{2V} \left[(2 - \alpha)(N_+^2 + N_-^2) - 2\alpha N_+ N_- \right. \\ &\quad \left. + 2N_0(N_+ + N_-) + 4\sqrt{\alpha} N_0 \sqrt{N_+ N_-} \cos \theta \right], \end{aligned} \quad (5)$$

where the reduced overlap factor is just given by $\alpha = V \int d^3r \eta_+ \eta_-$. Our model allows the α -factor to vary from one to zero, corresponding to the case that the two components are from thoroughly mixed to completely separated. For a homogeneous system, the gradient term H_0 can be dropped. The spin-irrelevant term H_u remains a constant since the total density distribution is hardly responsive to the evolution of domain structure [1, 15], $\rho(\mathbf{r}, t) \approx \rho(\mathbf{r}, 0)$. Thus the dynamics of domain formation is only determined by the Hamiltonian expressed in Eq. (5).

The Berkeley experiment considered a pure spinor condensate initially prepared in the unmagnetized state. It is important to emphasize that the total spin is conserved in an atomic quantum gas under experimental conditions [16, 17, 18]. Therefore the particle numbers of $M_F = 1$ and -1 component are always equal, $N_+ = N_-$. Such a system is described by the Hamiltonian

$$\mathcal{H}_s = -(1 - \alpha)(1 - n_0)^2 - 2n_0(1 - n_0) [1 + \sqrt{\alpha} \cos \theta]. \quad (6)$$

Here $\mathcal{H}_s = H_s / (N \frac{g_2}{2})$ with total particle number $N = N_+ + N_- + N_0$; $n_a = N_a / N$ is the fraction of $M_F = a$ component. n_0 and θ form a pair of conjugate variables

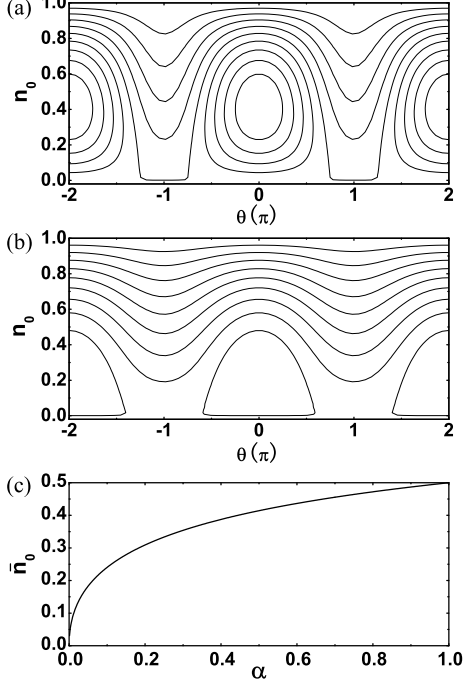


FIG. 2: Contour plot of the energy surface in the θ - n_0 plane with the overlap factor $\alpha = 0.5$ (a) and $\alpha = 0.1$ (b). (c) shows the fraction of $M_F = 0$ atoms at the minima, \bar{n}_0 , as a function of α .

and their equations of motion are given by

$$\frac{\partial}{\partial t} n_0 = -2\sqrt{\alpha} n_0 (1 - n_0) \sin \theta, \quad (7a)$$

$$\frac{\partial}{\partial t} \theta = 2(1 - \alpha)(1 - n_0) - 2(1 - 2n_0) [1 + \sqrt{\alpha} \cos \theta]. \quad (7b)$$

The population dynamics depicts the process of the domain formation; and the magnetization of magnetic domains is defined as $m = N_+ \eta_+ - N_- \eta_-$.

A number of theoretical works have investigated the spin dynamics of the ^{87}Rb spinor condensate [11, 12, 13]. We notice that previous theories usually treated the three components being mixed as they share the same spatial wave function, which is known as the single-mode approximation. Therefore the domain structure is smeared out and thus the ferromagnetic feature was not sufficiently dealt with. Those theories correspond the special case of $\alpha = 1$ in this letter.

The dynamical behaviors of Eqs. (7) can be visualized by the phase-space portrait with constant energy lines. Figure 2a and 2b plot the contour lines of energy with the overlap factor $\alpha = 0.5$ and 0.1 , respectively. In case of $\alpha = 1$, there are energy minima along $n_0 = 1/2$ at $\theta = 2n\pi$ [11, 12, 13] and all the contour lines are closed loops around those minima. As domains

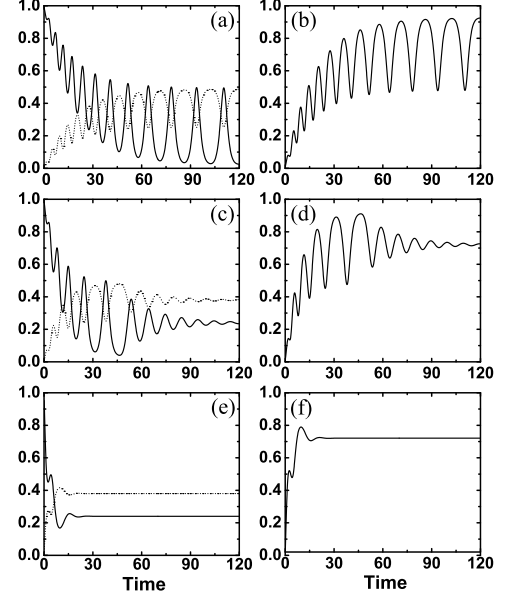


FIG. 3: Dynamics of domain formation for Bose systems with the overlap factor $\alpha = 0.1$ and $T_R = 20, 10$, and 2 from top to bottom. The left column shows the particle fractions of $M_F = 0$ (n_0 , solid lines) and $M_F = \pm 1$ (n_{\pm} , dotted lines) components. The right column shows the magnetization of magnetic domains, m . The initial state configuration is ($n_+ = 0.005, n_0 = 0.99, n_- = 0.005$).

build up, the value of α drops down and two apparent changes take place, seen in Fig. 2a and 2b. (i) There exist two distinct regimes in the phase space diagram. The newly appeared regime lies in the upper region of the figures, which consists of a set of open curves. Each line corresponds to a rotation type of solution, in which the relative phase θ is "running" with the time. The closed orbits, lying in the lower region, represent libration type of solutions, in which θ oscillates around the minimum. (ii) The positions of those minima move towards to smaller values of n_0 . The minima points are connected to the ground state of the system. Figure 2c shows the $M_F = 0$ particle fraction at the minima, \bar{n}_0 . The less α is, the smaller \bar{n}_0 is. It means that the domain formation tends to reduce the number of $M_F = 0$ atoms. This point has also been affirmed by Mur-Petit *et al.* who obtained a state with equipartition in populations, ($n_+ \approx 1/3, n_0 \approx 1/3, n_- \approx 1/3$), from a starting state (0.005, 0.99, 0.005) [14]. We derive that $n_0 = n_+ = n_- = 1/3$ when α drops to 0.25. In the limit case of $\alpha = 0$, $n_0 = 0$ and $n_+ = n_- = 0.5$.

The state of the quenched $M_F = 0$ condensate is viewed as a point lying in the upper region of the phase space diagram (Fig. 2a or 2c), when Eqs. (7) yield a self-trapping solution[19]. This motion reflects the quantum mechanical nature of the Bose-Einstein condensate. The

oscillating amplitude of n_0 is very small and the resulting magnetization m is so low in magnitude that it is hard to be probed experimentally during this stage[1]. The self-trapping effect prevents the growth of magnetic domains. This case is similar to the classical Larmor precession of a spin around magnetic field: it is rotating all the time, but the spin direction can be never parallel to the field without energy dissipation.

Then one has to take into consideration the effect of thermal agitation, which can change the energy of the system and drive the system into thermal equilibrium. Therefore, if n_0 departs from its thermal equilibrium value \bar{n}_0 , it will relaxes to \bar{n}_0 exponentially with a characteristic time scale T_R , called the relaxation time. Assuming that the relaxation velocity is proportional to $\bar{n}_0 - n_0$, $\partial\bar{n}_0/\partial t \propto \bar{n}_0 - n_0$, Eq. (7a) can be replaced with the following one [20],

$$\frac{\partial}{\partial t} n_0 = -2\sqrt{\alpha}n_0(1 - n_0)\sin\theta + \frac{\bar{n}_0 - n_0}{T_R}. \quad (8)$$

T_R scales qualitatively the thermal dissipation rate. Longer T_R denotes weaker thermal agitation.

Figure 3 displays the population of $M_F = 0$ and ± 1 components, as well as the magnetization m . The evolution of n_0 and m reflects the dynamical process of the domain formation. As shown in Fig. 3a and 3b, n_0 decreases *oscillatorily* with time driven by the thermal agitation, and meantime m arises. A very interesting result is that the oscillation amplitude *increases* as the system relaxes. Generally, the thermal agitation suppresses the macroscopic quantum coherence, and thus tends to kill oscillations of the population, while here we see that the oscillation is enhanced. Furthermore, the magnetization persists in oscillating for a very long period of time after the amplitude reaches its maximum. This result is qualitatively consistent with the Berkeley group's observation of the unstable magnetization mode[1].

If the thermal agitation gets stronger, the oscillation will be enhanced first, and then suppressed, as shown in Fig. 3c and 3d. Correspondingly, we divide the whole process into two periods with respect to the domain formation, the growing period and the stabilizing period. In the latter period, n_0 and m oscillate around their thermal equilibrium values and the amplitudes decrease gradually, then we have a stable domain structure eventually. If sketching the solution in the phase space diagram, one can find that the growing period is represented by the trajectory in the libration regime, and solutions for the stabilizing period lie in the rotation regime. Based on this understanding, Fig. 3a and 3b show only the growing period. Given the thermal agitation strong enough, the quantum mechanical feature will be smeared out in both periods, as Fig. 3e and 3f indicate.

According to the above discussions, the present model can qualitatively describe the dynamic process of the domain formation in a quenched $M_F = 0$ condensate. Sig-

nificantly, we show that the spatial separation of magnetic domains brings about much nontrivial effects on the spin dynamics of the ferromagnetic condensate. To get a quantitative description, more local details of the particle distribution and the relative phase should be considered.

In conclusion, we have investigated the dynamics of domain formation in a ferromagnetic spinor Bose-Einstein condensate, taking into account of the symmetry-breaking of the $M_F = 1$ and -1 components. Magnetic domains develop with the separation of $M_F = \pm 1$ components. Our results suggest that the $M_F = 0$ component in the condensate can significantly decay to a very small value, far less than $1/2$ as previous theories predicted. The domain structure is formed and stabilized with the help of the thermal dissipation. A thermally enhanced quantum-oscillation is observed during the process.

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